

# Quantifying Non-trivial Open-Ended Evolution Reveals Necessary and Sufficient Conditions

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## Abstract

One of the most remarkable features over the last  $\sim 3.5$  billion years of life on Earth is the apparent trend of innovation and open-ended growth of complexity. However, this trend does not have a satisfactory explanation in terms of currently-known principles. Here, we demonstrate that a variant of CA are capable of open-ended evolution and innovation by implementing state-dependent dynamical rules. To quantitatively evaluate potential for open-ended evolution and innovation, we present formal definitions of *open-ended evolution* as patterns that are non-repeating within the expected Poincaré recurrence time of an equivalent isolated system, and of *innovation* as trajectories not observed in isolated systems. We show that a small subset of state-dependent systems satisfy both definitions. We compare these systems to a set of controls including CA evolved according to fixed rules and randomly evolved rules and show that state-dependent systems are statistically more reliable at producing both open-ended evolution and innovation. We further show how state-dependent CA allow for sustained growth of complexity, demonstrating that both the complexity and percentage of open-ended cases increases with increasing environment size. Our results indicate that uncovering the principles governing open-ended evolution and innovation in the biosphere will likely require removing the segregation of states and (fixed) dynamic laws characteristic of the physical sciences in attempts to model biological complexity.

## Motivation

We question whether or not open-ended evolution is possible *at all* in a physical system. Open-endedness can easily be achieved through mechanisms like the Busy Beaver problem [1] or by invoking randomness [2] either observing an ever-increasing trend in complexity or by observing an endless diversity of innovation emerge. Other mechanisms such as counting to infinity, air moving around in a room, elementary cellular automata rule 30 on an infinite lattice, and Turing machines are all examples of open-endedness that is generated by trivial mechanisms. Trivial open-endedness demonstrates a weakness in scalability, since it is increasingly difficult to produce systems that are differentiable from random or reproducible at large scales.

Therein remains the question of bounded and fully-deterministic systems, finite in their size and unable to ac-

commodate a truly infinite landscape of possibilities. Computational resources that simulate biological systems are faced with the problem of being confined to a finite space. Is open-ended evolution possible in these types of systems? The amount in complexity will eventually reach a limit in such a space, thus excluding the possibility for a finite space to achieve an ever-increasing trend in complexity. However, this limitation does not discount the notion of open-ended evolution in a finite system altogether. In fact, the possibility for continual innovation allows the possibility to exist, even if the system maintains a maximal amount of complexity over time.

Our intention is to explore new non-trivial mechanisms that generate open-ended evolution in bounded, deterministic systems. We accomplish this by first introducing definitions of innovation and open-ended evolution that exclude trivial mechanisms and agree with general intuition about open-endedness, then generate mechanisms that produce robust, scalable open-ended evolution.

## Open-Ended Evolution

Open-ended evolution cannot be solved with a fixed physical local law in a bounded, deterministic system that is completely closed to any outside influence. If the system is completely open, such as invoking randomness, then open-ended evolution is trivial and non-robust as mentioned previously. Thus, robust, non-trivial open-ended evolution is only possible in a system that is somewhere between completely closed and completely open.

We consider definitions that are applicable to any such universe  $u$  that can be decomposed in two (interacting) subsystems  $o$  and  $e$  (nominally the “organism” and “environment”). Since the notion of innovation is not entrained to a single fixed input and physical law, a notion of comparison must be embedded within its definition:

**Definition 1 Innovation:** *A system  $u$ , which can be decomposed into subsystems  $o \in O$  and  $e \in E$  that interact according to a function  $f$ , exhibits innovation if there exists a  $t_r$  such that  $f^{t_r}(o) = \{s_o(t_1), s_o(t_2), s_o(t_3) \dots s_o(t_r)\}$  is not contained in the set of all possible state-trajectories*

$\{s_o\}_I$  for an isolated (non-interacting) system  $o \in O$ .

That is to say a subsystem is capable of innovation if its behavior is *not* contained in the dynamics of *any* isolated system of an equivalent size. Likewise, we define:

**Definition 2 Open-ended evolution:** A system  $u$ , which can be decomposed into subsystems  $o \in O$  and  $e \in E$  that interact according to a function  $f$ , exhibits open-ended evolution if there exists a  $t_r$  such that  $f^{t_r}(o) = \{s_o(t_1), s_o(t_2), s_o(t_3) \dots s_o(t_r)\}$  or  $f^{t_r}(o) = \{r_o(t_1), r_o(t_2), r_o(t_3) \dots r_o(t_r)\}$  is non-repeating for  $t_r > T_P$ , where  $T_P$  is the Poincaré recurrence time for an isolated system  $o \in O$ .

A universe  $u$  exhibits open-ended evolution (OEE) if, and only if, the behavior of its states or the physical laws that govern the subsystem (the “organism”) is non-repeating within the expected Poincaré recurrence time  $T_P$ .

We contend that *non-trivial* open-endedness must satisfy both Definitions 1 and 2. For example, it should be immediately clear from Definition 2 that *any* non-innovative behavior cannot be open-ended. Furthermore, the trivial examples mentioned earlier are excluded from satisfying both definitions. For example, some non-bounded systems could in principle satisfy Definition 2 but not Definition 1, since their dynamics are equivalent to an isolated system (they are not innovative). In order to meet both definitions, a system must be embedded in a larger universe by having semi-permeable boundaries.

For deterministic systems, open-ended evolution can only ever be an attribute of a *subsystem* and not globally, as the full system  $u$  will always be limited by the Poincaré recurrence theorem. This conceptual step might seem trivial, but here we *must* emphasize the partition of subsystems and their relative timescales to the global system in which they are embedded. We suggest the timescale of a subsystem embedded in global dynamics operates on a timescale that is identical to an isolated system of the same size. Open-endedness is only realized when timescales are apportioned according to their isolated equivalents.

Without loss of generality, we test these definitions in a modified one-dimensional cellular automata (CA) universe that is capable of demonstrating fully open systems, fully closed systems, and systems that are in-between. Since CA are a common model for complex systems, they are well-studied and provide a tractable means for applying our definitions.

## Experiment

While traditional elementary cellular automata evolve according to a fixed dynamical rule, the three CA variants presented here evolve according to explicitly *time-dependent* rules, where the time dependence takes on different functional forms according to how open the system is. The first (Case I) evolves rules deterministically as a function

of the current state of the entire system thereby implementing state-dependent rules, and thus is self-referential. Case I CA is an example of a system that has semi-permeable boundaries since it depends on its own state and the state of a second, separate CA. The second type (Case II) allows the rules to evolve deterministically as a function of time, but this evolution is not a function of the state. It is completely open to driven changes by allowing the rule to change as a function of a second, separate CA only. In the final variant (Case III), rule evolution is stochastic. Like Case II CA, it is completely open, but the changes are not driven by another external CA.

To quantify the presence of open-ended evolution in each system, we measure the time it takes for a system to repeat in both the rules and the states. If this time is greater than the Poincaré recurrence time for an equivalent, isolated system (Definition 2) while also being innovative (Definition 1), the system is considered to exhibit open-ended evolution. In addition, we measured the compressibility of such system dynamics and their sensitivity to initial conditions although our definitions do not depend on these complexity measures.

## Results

For the smaller system sizes explored in depth, the majority of all executions of all three CA variants were innovative by Definition 1. In addition, the percent of organisms that were found to be innovative increased as a function of CA size. The fraction of OEE cases observed in our statistical samples for each size confirms that indeed larger environments are much more robust generators of a larger fraction of OEE cases for organisms of fixed width.

It is worth mentioning that satisfying the criterion for open-ended evolution, per Definition 2, necessitates that complex systems process information on different time scales. This does not suggest that closed, deterministic systems will never repeat, as often posed as an argument against the possibility of attaining OEE in a fully deterministic (closed) universe. Instead, it suggests that complex systems operate on multiple spatial and temporal scales, and OEE is possible for some of these spatiotemporal scales through deterministic and bounded processes. The results therefore connect two hallmarks of life, by demonstrating that self-reference may be explicitly linked to a robust mechanism for generating OEE. Both hallmarks can emerge from simple rules whose dynamics are otherwise unsurprising without multiple layers of information processing.

## References

- [1] Hector Zenil, *A Computable Universe*, World Scientific Publishing, 2013.
- [2] Nicholas Guttenberg and Nigel Goldenfeld, *Cascade of Complexity in Evolving Predator-Prey Dynamics*, Physical Review Letters, 100, 058102, 2008.